

New Numerical Integration Routine for the Nonorthogonal PEEC Approach

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This contribution focuses on special aspects regarding numerical integration routines for nonorthogonal PEEC cells. By using averaged orthogonal subelements in the numerical integration routine, the slow convergence caused by the singularities is avoided and consequently, a fast evaluation of the self terms is enabled. The approach is verified by a spiral planar coil with a wire of circular cross section. Here, the current density is computed by the proposed algorithm and compared with FEM results showing a good agreement.

Index Terms—Non-orthogonal partial element equivalent circuit (PEEC) method, filament mutual inductance, numerical integration, partial element computation.

I. INTRODUCTION

FOR developing efficient power electronic systems, the use of computational simulation tools is getting more and more important since higher efficiencies, less construction space and reduced hardware costs are demanded. This is especially true for conductors and coils needed in inductive components such as transformers or chokes. A well-known approach to predict effects e. g. the skin- or proximity effects in such coils or conductors is the partial element equivalent circuit (PEEC) method. In this terminology, the discretized geometric coupling elements can be interpreted as partial network elements and the electromagnetic system can be solved via any SPICE compatible solver.

II. PARTIAL NETWORK ELEMENTS OF NONORTHOGONAL MQS-PEEC FORMULATION

The partial resistances R_{mm} and inductances L_{mn} depend on the chosen basis and testing functions. In the orthogonal brick-shaped case where in each cell a constant current density is assumed formulations for the partial elements are given in e. g. [1], [2]. In contrast in the nonorthogonal case, a local coordinate system (a, b, c) in each current cell is defined [3] and its metric coefficients h_i and unit vectors \vec{e}_i with $i = a, b, c$ have to be considered in the basis and testing functions. The resulting partial inductance is given in e. g. [4] and is repeated here as

$$L_{mn} = \frac{\mu_0}{4\pi 16} \int_a^b \int_b^c \int_c^a \int_{a'}^{b'} \int_{b'}^{c'} \int_{c'}^a \frac{1}{|\vec{r} - \vec{r}'|} h_{a,m} h'_{a,n} (\vec{e}_{a,m} \cdot \vec{e}'_{a,n}) da' db' dc' da db dc. \quad (1)$$

For practical applications, the six-fold integrals in (1) have to be evaluated according to the specific geometry. For the orthogonal formulation, exact analytical solutions exist for special arrangements. For all other cases various approximation techniques can be applied, e. g. the solution with a filamentary approach given in [5] and introduced as L_{filament} . A more

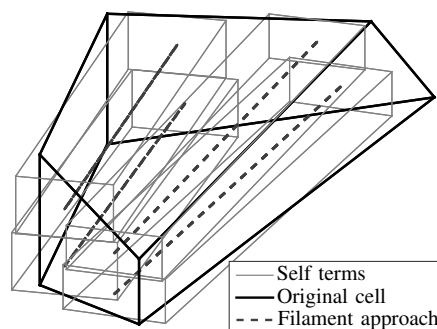


Fig. 1. Mixed self inductance calculation with approximated self terms and the filament approach for an integration order of two using (2)

accurate alternative is to evaluate the integrals with a numerical integration technique, which is especially important for the nonorthogonal case.

It should also be noted that the number of partial inductances L_{mn} grows with the square of the number of cells because all cells couple with each other.

III. SPECIALIZED NUMERICAL INTEGRATION ALGORITHM FOR NONORTHOGONAL PARTIAL INDUCTANCES

Because the number of computations of (1) exhibit quadratic growth, these calculations are often the bottleneck of the overall computation time and it is worth of considering the six-fold numerical integration routine in detail. At first, it is convenient to reduce the six-fold integral of (1) to a four-fold integral [6] identifying the kernel as being the solution of arbitrarily oriented current filaments for which analytical solutions exist given as L_{filament} .

By doing so, special attention has to be paid to the singularities of the kernel at $\vec{r} = \vec{r}'$ in (1). Every time, when $b = b'$ and $c = c'$ in the nonorthogonal coordinate system, the analytical solution of the kernel L_{filament} becomes singular. In order to avoid this situation, the filamentary approach L_{filament} is

substituted by the analytical solution of the self inductance of a orthogonal current cell $L_{\text{self,exct}}$ given in [1] with an averaged thickness and width of this singularity determined under the assumption that the volume of the nonorthogonal subparts and the volume of averaged orthogonal cells is equal. Applying the proposed approaches (1) becomes

$$L_{mm} \approx \left[\int_b \int_{b'} \int_c \int_{c'} L_{\text{filament}} db' dc' db dc \right]^{b \neq b', c \neq c'} + L_{\text{self,exct}}(b = b', c = c'). \quad (2)$$

The illustration of this mixed form of computed coupled filaments with a replacement of the singularities is shown in Fig. 1. For visualization aspects, the integration order for the self inductance has been chosen to be two. The mutual couplings are computed with the dashed filaments whereas the self terms of the sub-elements are computed via the analytical solution of the grey highlighted orthogonal bricks.

Although this approach can lead to overlapping volumes, the advantage of the proposed algorithm is the fact that more accurate results are obtained in the sense that the integration order and thus numerical costs needed for a specific accuracy can be reduced compared to a basic approach, where the singularities are simply substituted by zero. It is also worth mentioning, that this approach is not only valid for inductive cells but also for nonorthogonal capacitive cells in a straightforward manner.

IV. NUMERICAL EXAMPLE OF A SPIRAL PLANAR COIL

For verifying the introduced aspects of the nonorthogonal PEEC method and the fast integration technique, a spiral planar coil with three turns having a wire of circular cross section is analyzed. The geometry of the setup can be inspected in Fig. 2(a). Here, the inner radius is 20 mm, the wire radius 0.5 mm and the spacing between the turns 1 mm. The original circular outer contour of the coil is approximated by a polygon with 12 edges and 4680 long and thin nonorthogonal PEEC cells. For this geometrical setup using a frequency of 100 kHz typically effects like skin- and proximity effects are expected and can be verified. Because there exists no known analytical expressions of the calculated current distribution and impedance of the coil for arbitrary parameter settings, a qualitative comparison of the current distribution is focused on. As a reference solution, a finite element method (FEM) simulation is run with a commercial 3D full-wave solver. Via postprocessing, the absolute value of the current density is computed and shown in a cross section of the coil in Fig. 2(c). For comparison aspects, the same is done with the nonorthogonal PEEC approach in Fig. 2(b). It can be seen from the figures that both results agree very well in terms of the form of the current distribution resulting from skin- and proximity effects. The total computation time using the proposed nonorthogonal PEEC approach for the given example is 185 s on a 2.7 GHz processor and 120 GB of RAM. The FEM results in Fig. 2(c) are calculated with 4264 113 first order cells with a total computation time of 1612 s on the same hardware setup resulting in a speedup of the PEEC approach of approximately 10. The multitude of the FEM cells is needed

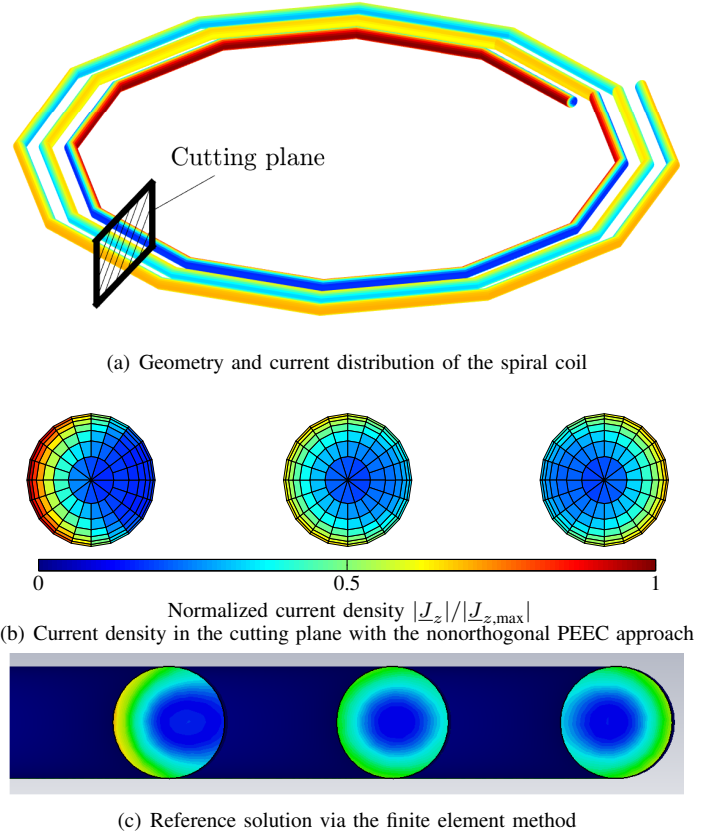


Fig. 2. Calculated current distribution of the 3D example coil

to correctly capture the influence of the eddy current effects inside the conductors.

V. CONCLUSION

Throughout this paper, a fast numerical integration technique for the nonorthogonal PEEC formulation is proposed and evaluated with a practical application of a circular cross sectioned conductor, which is discretized using nonorthogonal current cells. In the final distribution more details about the proposed algorithm will be given, a multifunction PEEC approach will be proposed and an academic example of a 2D conductor with a circular cross section will be evaluated.

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